Effect of Time Delay Signature on Reservoir Computing using Optical Feedback Semiconductor Lasers

Project Number: 113-2813-C-006-159-E

Long-Gui He[1], Chin-Hao Tseng[1], Sheng-Kwang Hwang[1,2,*]

1. Department of Photonics, National Cheng Kung University, Tainan, Taiwan

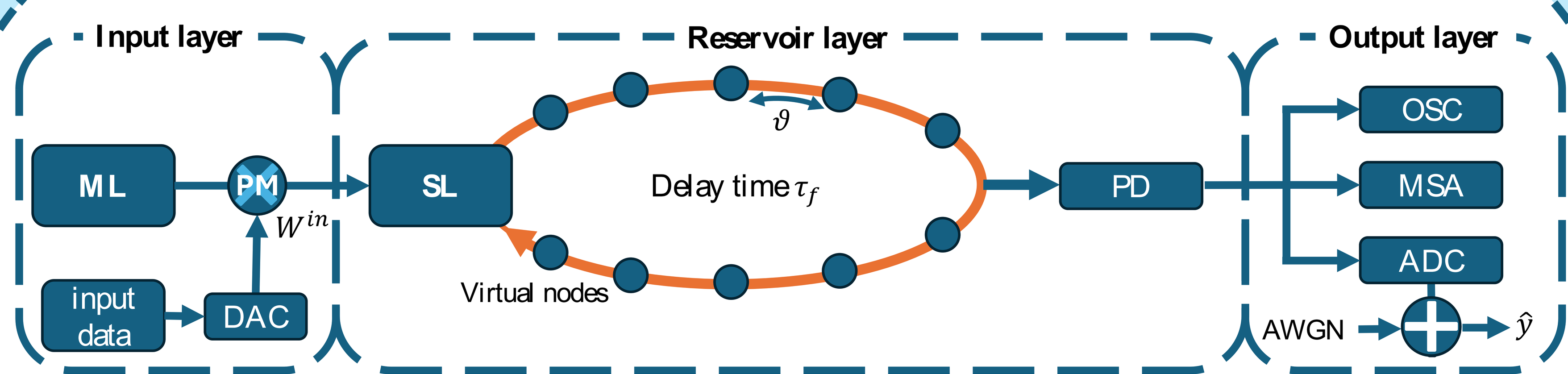
2. Meta-nanoPhotonics Center, National Cheng Kung University, Tainan, Taiwan

**skhwang@mail.ncku.edu.tw*

NSTC 國家科學及技術委員會 National Science and Technology Council

We demonstrate a direct correlation between the prediction accuracy of an OFSL-based Photonic Reservoir Computing and the suppression of its time-delay signature. By systematically analyzing the chaotic dynamics and the performance on a forecasting task, we show that minimizing the signal's self-correlation is crucial for optimizing the system. This provides a clear guideline for enhancing the performance of such computational systems.

Simulation Setup



Input layer: The data u(t) modulates the phase of the injection field from a Master Laser (ML) via a Phase Modulator (PM) before it enters the Slave Laser (SL).

Reservoir layer: Composed of a Slave Laser (SL) with delayed optical feedback. Its chaotic dynamics serve as the computational reservoir for processing data.

Output layer: The SL's output intensity is analyzed in the time domain (OSC) and frequency domain (MSA). After digitization (ADC), the signal is used to train the output weights to get the final prediction.

Rate Equation

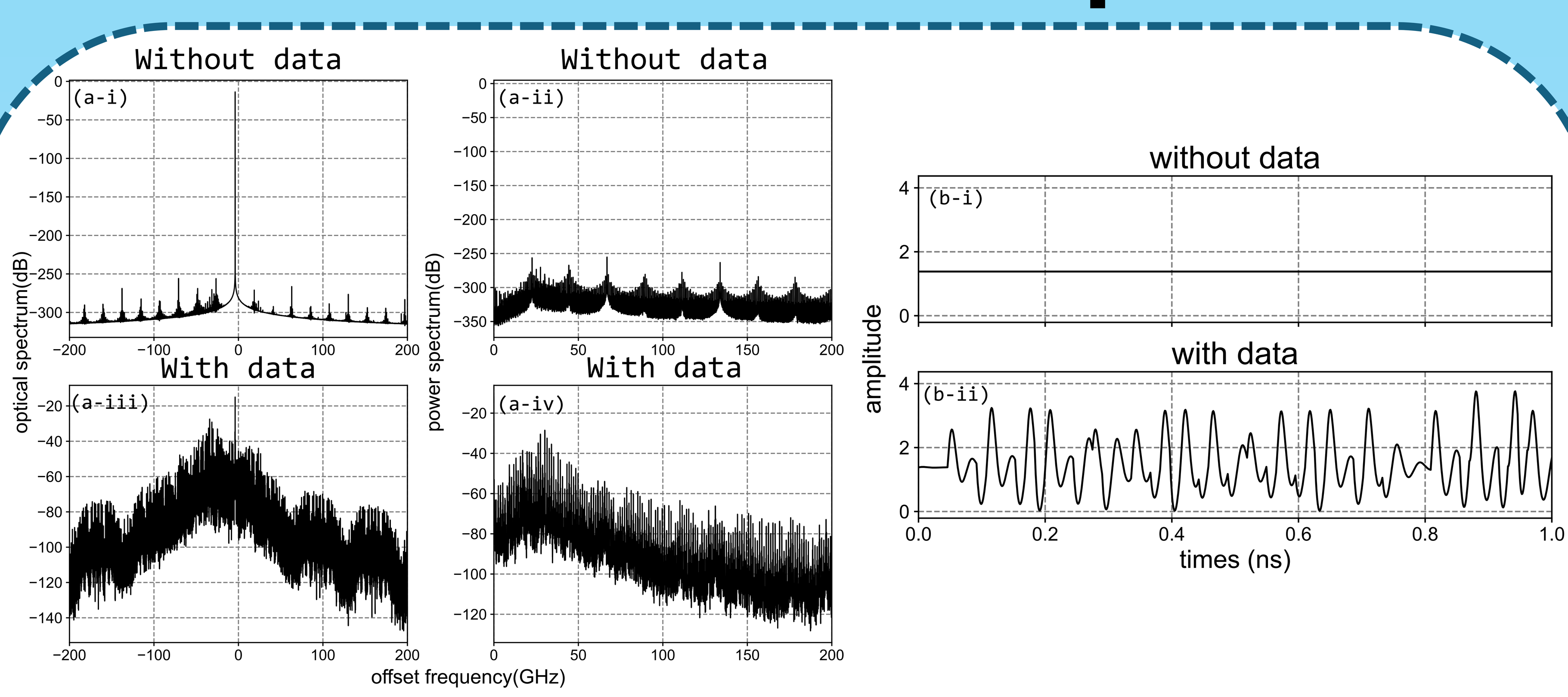
$$\frac{da_r}{dt} = \frac{1}{2}G(a_r + ba_i) + \xi_i\gamma_c[\cos(\Omega_i t - \pi u(t)) + \sin(\Omega_i t - \pi u(t))] + \xi_f\gamma_c a_r(t-\tau_f)$$

$$\frac{da_i}{dt} = \frac{1}{2}G(a_i - ba_r) + \xi_i\gamma_c[\cos(\Omega_i t - \pi u(t)) - \sin(\Omega_i t - \pi u(t))] + \xi_f\gamma_c a_i(t-\tau_f)$$

Key Parameters :

- $\Omega_i = 2\pi f_d/\gamma_c$: Normalized frequency detuning
- ξ_i : Normalized injection strength
- ξ_f : Normalized feedback strength
- τ_f : Feedback delay time

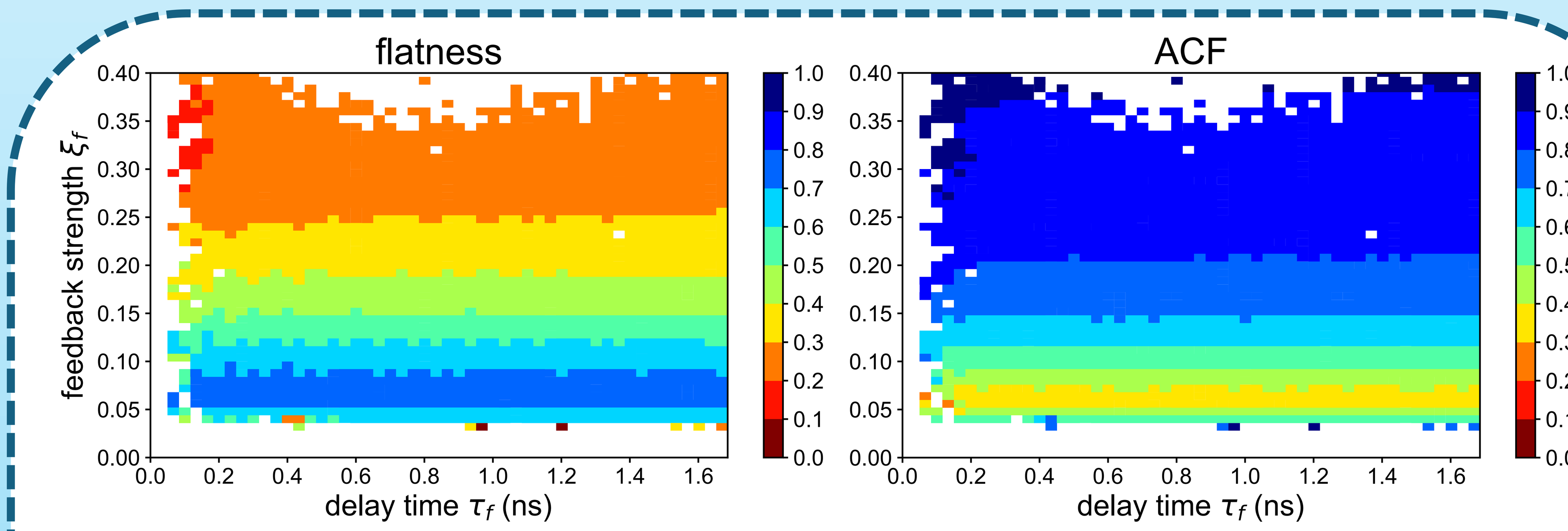
MSA & OSC output



The figure on the right shows the 1ns **time series** of the laser operating in a stable region. Without data (b-i), the laser dynamic is calm; however, after data is injected (b-ii), the reservoir is driven by the training data and shows a **rich chaotic dynamic response**.

Similarly, the **optical and power spectra** on the left show the same characteristics. After data is injected (a-iii, a-iv), the single-frequency spectrum is converted into a broadband chaotic spectrum. This indicates that the reservoir's dynamic has become more complex, enabling it to map the input data into a **high-dimensional space**.

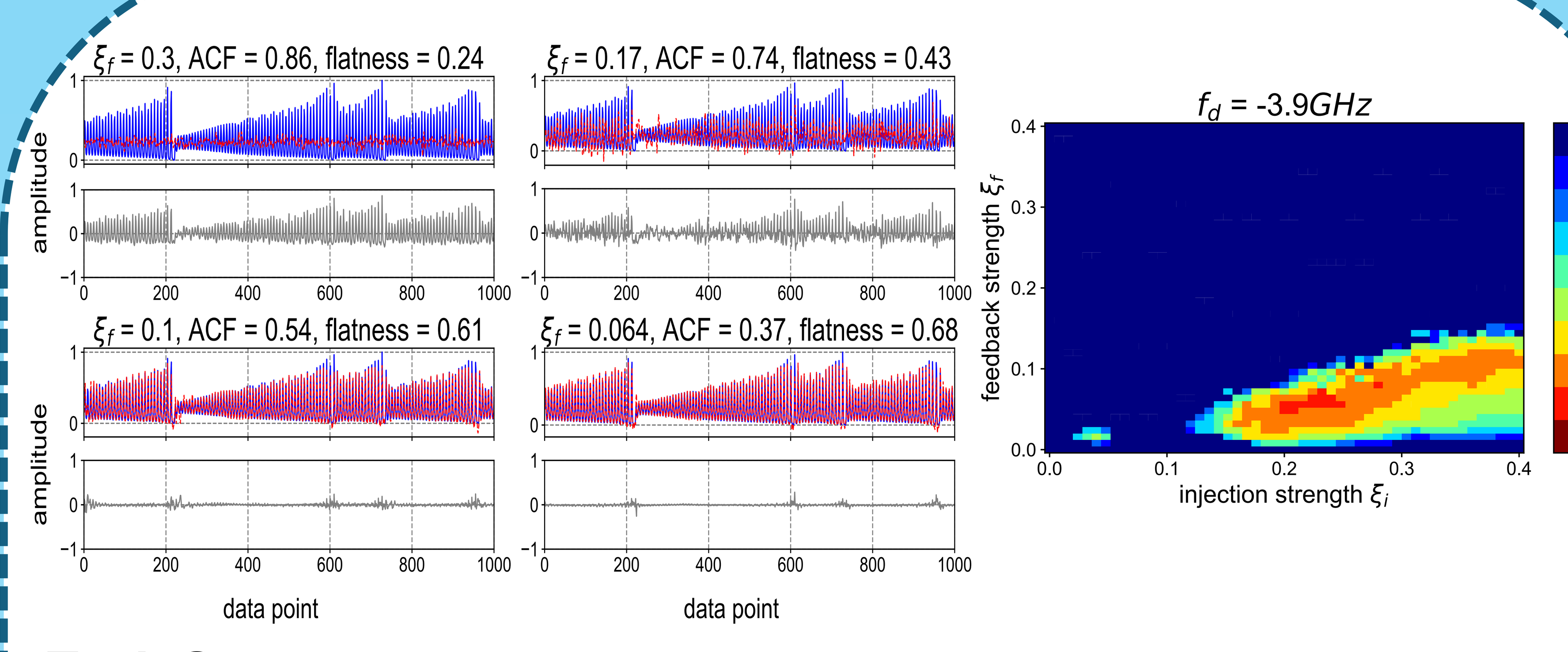
Analysis of the Reservoir



The quality of the chaotic reservoir determines the system's prediction performance. We use two metrics to characterize the chaos **without data injection**:

- **ACF (Autocorrelation Function):** Measures periodicity. **Lower ACF** is better.
- **Flatness:** Measures spectral complexity. **Higher flatness** is better.
- **Key Finding:** Increasing ξ_f significantly degrades the chaos quality by increasing the ACF and reducing flatness.
- **Next Step**: Based on this analysis, we selected four distinct ξ_f values (0.064, 0.1, 0.17, 0.3) to investigate their impact on prediction accuracy.

Results of Prediction Task



Task Setup:

•Performed a one-step-ahead prediction on the **Santa Fe chaotic time-series**.

•The system used 50 virtual nodes, with 3000 training points and 1000 testing points

Finding 1: Lower Feedback Strength (ξ_f) Improves Accuracy

The time-series plots show that lower ξ_f leads to higher prediction accuracy (lower error).

Best Performance: An NMSE of **0.017** is achieved at **ξ_f= 0.064.** This operating point corresponds to the best chaotic properties (lowest ACF and highest flatness) from our previous analysis.

This confirms that the randomness of the reservoir is a key performance factor.

Finding 2: Optimal Operating Region

The 2D map reveals the optimal operating region is around $\xi_i \approx 0.24$ and $\xi_f \approx 0.064$.

A key trend is observed: a higher injection strength (ξ_i) requires a higher feedback strength (ξ_f) for optimal performance.

Reason: Stronger feedback is needed to maintain the reservoir's dynamic richness against the chaos suppression caused by strong injection.

This study confirms that adopting an operating point with a **low Time Delay Signature (TDS)** can significantly improve training results. By properly adjusting parameters, the RC system can achieve a **lower Normalized Mean Square Error (NMSE)**. This indicates that **lower self-correlation and higher spectral flatness** provide richer data features, which in turn improves prediction accuracy. This research provides a **theoretical basis for high-efficiency RC applications** and offers new ideas for **communication systems and high-speed data processing**.

Future work: could explore the optimization of the node time (ϑ). This is particularly important as the relaxation resonance frequency shifts with injection strength, and dynamically adapting ϑ could be crucial for developing robust, high-speed photonic RC systems.